Superconductivity in homologous cuprate series: deep oscillations of pairing Coulomb potential

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Kinematic constraint arising in the case of superconducting pairing with large momentum results in a cutoff of the screened Coulomb potential excluding large momentum transfers. This leads to a pairing potential oscillating in the real space that ensures a rise of bound singlet pairs. In multilayer cuprates, there is strong Coulomb interaction between particles composing the pair not only in the same cuprate layer but in the neighboring layers as well. In the framework of such a scenario, we explain a universal dependence of the superconducting transition temperature on the number of layers in the unit cell observed in homologous cuprate families.

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1. All families of superconducting (SC) cuprate compounds investigated manifest a striking dependence of the transition temperature T_c on the number n of CuO_2 planes in the unit cell. When n increases, the function $T_c(n)$ grows at first, then, after passing the maximum at $n = 3$, decreases monotonically.[1] This behavior is shown schematically in fig.1 which presents $T_c(n)$ for a homologous series of mercurocuprates.[2] One can consider an explanation of such a dependence, reflecting a fundamental mechanism of SC pairing in the cuprates, as one of the most important problems of high-T_c superconductors.[3]

A distribution of doped charge turns out to be nonhomogeneous in multilayer compounds: the inner layers in the unit cell are underdoped as compared with the outer layers to ensure the minimum of the electrostatic energy.[4] Thus, in optimally (on the average) doped multilayer compound, inner (outer) cuprate planes are underdoped (overdoped) in comparison with the optimal doped compound of the family with a single cuprate plane in the unit cell.

Weak coupling of the neighboring cuprate layers due to coherent tunnelling of the pairs[5] leads to extremely weak initial rise in the function $T_c(n)$ and its further saturation at $n > 3$. To explain the observed decrease of $T_c(n)$ at $n > 3$, Chakravarty et al.[5] took into account a nonhomogeneity of carrier distribution in the unit cell and also a competition between the SC and insulating (orbital current d - symmetry density wave[6]) orders.

A considerable rise in the SC transition temperature with an increase of the number of cuprate layers in the unit cell can be associated with the fact that a spatial scale of the pairing interaction exceeds the spacing between the neighboring layers. In contrast to many models based on the extremely localized (in the real space) SC pairing interaction, such a feature is inherent in the pairing with large momentum under screened Coulomb repulsion (η_K - pairing).[7]

2. Screening of Coulomb repulsion in classic electron gas results in the fact that the interaction energy of two particles at a distance r takes the form $U(r) = (e^2/r)\exp(-r/r_0)$ corresponding to the Fourier transform $U(k) = 4\pi e^2/(k^2 + k_0^2)$. Here, $r_0 = k_0^{-1}$ has the meaning of Debye screening length. The Thomas-Fermi approximation leads to the same expression for the interaction energy also in the case of degenerate electron gas with the screening length $r_0 = (4\pi e^2 ng)^{1/2}$, where n is the concentration and g is the density of states on the Fermi level. It should be noted that, when one takes into account the step-wise electron distribution in the momentum space, the screened Coulomb potential turns out to be modified qualitatively,[8]

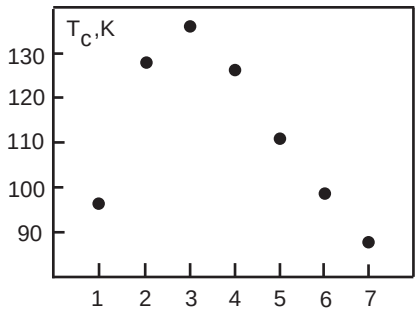

Рис. 1: SC transition temperature (T_c) of optimal doped compound of the family $HgBa_2Ca_{n-1}Cu_nO_{2n+2+\delta}$ vs. a number n of cuprate layers in the unit cell (according to Ref.[2]).

$$U(k) = 4\pi e^2/[k^2 + k_0^2 f(k/2k_F)]. \qquad (1)$$

Here, k_F is the Fermi momentum and the Lindhardt function has the form

$$f(x) = \frac{1}{2}\left(1 + \frac{1-x^2}{4x}\ln\left|\frac{1+x}{1-x}\right|\right). \qquad (2)$$

The weak singularity of the potential (1) at $k = 2k_F$ corresponding to the point of contact of two shifted Fermi surfaces leads to the damped Friedel oscillations (with the wavelength π/k_F) of the screened Coulomb potential. The latter at large r has the form

$$U(r) \simeq \frac{e^2}{2\pi}\frac{\cos 2k_F r}{r^3}. \qquad (3)$$

In the case of the nested Fermi surface, the singularity turns out to be enhanced that can result in a rise of

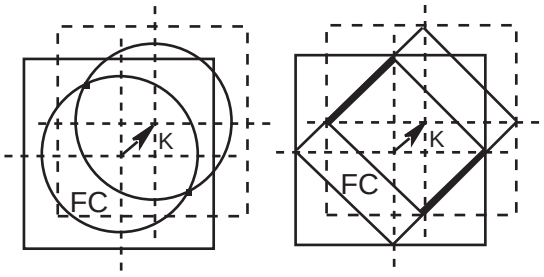

Рис. 2: A rise of the domain of kinematic constraint in the case of η_K - pairing in 2D electron system for isotropic (left) and nested FC (right); the mirror nesting condition is fulfilled on the heightened pieces of the FC.

charge or spin density waves (sctructural phase transition or antiferromagnetic state, respectively).

Thus, there is a region of the real space where the screened repulsive Coulomb potential becomes negative. This is sufficient to ensure SC pairing with non-zero angular momentum of the relative motion of the pair with zero total momentum.[9] However, due to a weakness of the Kohn singularity, the SC transition temperature turns out to be very low.[9]

3. A rise of a domain Ξ of kinematic constraint[7] in the case of SC pairing with large total momentum $\boldsymbol{K}$ leads to an enhancement of the singularity of the pairing potential $U(\boldsymbol{k}-\boldsymbol{k}')$. Indeed, since in the self-consistency equation that determines the energy gap parameter $\Delta(\boldsymbol{k})$ the momenta of the relative motion $\boldsymbol{k}$ and $\mathbf{k}'$ are defined (at $T=0$) only inside a finite region Ξ of the momentum space, there is a cutoff of far Fourier components of the potential $U(\boldsymbol{k}-\boldsymbol{k}')$ on the boundary of this region. It should be noted that, in a general case, the kinetic energy of the pair at $\boldsymbol{K}\neq 0$ vanishes only at some points inside Ξ. Therefore, because of the fact that a logarithmic singularity in the right-hand side of the self-consistency equation becomes eliminated, a non-trivial solution to this equation cannot exist at $U\to 0$.

We show in fig.2 how the domain of kinematic constraint arises in the case of isotropic two-dimensional (2D) dispersion. At $\boldsymbol{K}=0$, the pair state is formed by all one-particle (electron and hole) states inside the Brillouin zone and the kinetic energy of the pair vanishes on the whole Fermi surface (the Fermi contour (FC), in the case of 2D electron system). When the total momentum $\boldsymbol{K}$ of the pair increases, the region Ξ that contributes to the state of the pair decreases down to zero at $K=2k_F$. At $0<K<2k_F$ the kinetic energy of the pair vanishes at two points only: these points separate the regions of the momentum space with electron and hole filling. In such a case, the logarithmic singularity in the self-consistency equation is absent.

The FC with mirror nesting feature,[7] corresponding to the coincidence of the energies $\varepsilon(\boldsymbol{K}/2+\boldsymbol{k})$ and $\varepsilon(\boldsymbol{K}/2-\boldsymbol{k})$ of the particles composing the pair on a finite piece of the FC, results in the fact that the kinetic energy of the pair vanishes not at the isolated points but on a finite line inside Ξ. It means that, under mirror nesting condition, the logarithmic singularity in the self-consistency equation survives and the non-trivial solution exists at $U\to 0$. The effective coupling constant in the exponent of the gap parameter is proportional to the length of the pieces of the FC on which the mirror nesting condition is fulfilled,

$$\varepsilon(\boldsymbol{K}/2+\boldsymbol{k})=\varepsilon(\boldsymbol{K}/2-\boldsymbol{k}). \qquad (4)$$

The square FC corresponding to the half-filled energy band (fig.2) manifests apparent mirror nesting for any pair momentum $\boldsymbol{K}$ directed along one of the diagonals of the Brillouin zone under the condition that $K<K_\pi$ where $\boldsymbol{K}_\pi=(\pi,\pi)$. When $\boldsymbol{K}$ increases, the length of the pieces of the FC (on the sides parallel to $\boldsymbol{K}$) corresponding to the mirror nesting, decreases down to zero at $\boldsymbol{K}=\boldsymbol{K}_\pi$. Note that at $\boldsymbol{K}=\boldsymbol{K}_\pi$ there is a conventional nesting (for the opposite sides of the FC), $\varepsilon(\boldsymbol{p}+\boldsymbol{K}_\pi)=-\varepsilon(-\boldsymbol{p})$, resulting in the insulating state in the case of the parent compound. The insulating gap $2\Delta_s$ arises at the position of the FC which becomes a boundary of the magnetic Brillouin zone of 2D structure with long-range spin antiferromagnetic (AF) order.

4. Thus, the half-filled band of the parent compound with spin AF order turns out to be split into two subbabds so that the FC of doped compound appears as small hole pockets[10] near the top of the lower subband. The parts of the pockets situated in the first magnetic Brillouin zone of the parent compound (main bands) form the FC with maximum spectral weight of photoemission. The other parts of the pockets (shadow bands[11] in the second magnetic zone) correspond to the considerably lower spectral weight decreasing with doping together with $2\Delta_s$. Each pocket manifests perfect mirror nesting for SC pairs with the momentum $\boldsymbol{K}_\pi$. In this case, the momentum of one of the particles that compose the pair belongs to the main band whereas the momentum of the other particle belongs to the shadow band of the same pocket.

In the case of two pockets situated along the diagonal of 2D Brillouin zone, there is a perfect conventional nesting with the same momentum $\boldsymbol{K}_\pi$. The momenta of the particle and the hole composing the electron-hole pair belong to the different bands of the pockets: if one of the components of the pair is related to the main band the other one should be associated with the shadow band. The FC with both conventional and mirror nesting features can ensure a competition or coexistence of the SC and insulating (different from the spin amtiferromagnet) ordered states.

The η_K - pairing channel is efficient under mirror nesting of the FC. In the case of hole pockets, such a condition is fulfilled for each of the crystal equivalent momenta $\boldsymbol{K}_\pi=(\pm\pi,\pm\pi)$ with their own domains of kinematic constraint Ξ.

To take into account the fact that one of the particles of the pair belongs to the main band of the hole pocket and the other one is related to the shadow band, we

represent the pairing interaction in the form

$$U_{\mu\mu'}(\boldsymbol{k},\boldsymbol{k}') = \tilde{v}_{\mu}(\boldsymbol{k})U(\boldsymbol{k}-\boldsymbol{k}')\tilde{u}_{\mu'}(\boldsymbol{k}'), \qquad (5)$$

where μ enumerates the layer in the unit cell. Here, $U(\boldsymbol{k}-\boldsymbol{k}')$ is the Fourier transform of the screened Coulomb interaction, $\tilde{v}_{\mu}(\boldsymbol{k})$ and $\tilde{u}_{\mu'}(\boldsymbol{k}')$ are the coefficients of the Bogoliubov transformation that diagonalize the Hamiltonian describing the electron-hole pairing with a rise of spin AF order; note that $\tilde{v}_{\mu}(\boldsymbol{k}+\boldsymbol{K}) \to \tilde{u}_{\mu}(\boldsymbol{k})$.[12] An extension of the pockets with doping is accompanied with a decrease in the insulating gap $2\Delta_s$ and a strong depression (due to a deviation from the half-filling) of the spectral weight of the shadow band $\sim u^2_{\mu}(\boldsymbol{k})$. This leads to the effective restriction (in the momentum space) to the pairing interaction.

5. This can be considered as a reason to replace the kernel $U(\boldsymbol{k}-\boldsymbol{k}')$ of the pairing interaction operator by the approximate degenerate kernel

$$U(\kappa) = U_0 r_0^d (1-\kappa^2 r_0^2/2), \qquad (6)$$

presenting two terms of the expansion of the true kernel into the power series. Here $d = 2(3)$ for the two(three)-dimensional system, U_0 and r_0 have the meaning of the characteristic Coulomb energy and the screening length, respectively. The potential (6) manifests itself at small momentum transfer $\kappa \equiv |\mathbf{k}-\mathbf{k}'|$. Thus, there is an effective cutoff of the Coulomb repulsion at momenta that are less than the value k_c which can be considered as a characteristic length of the domain of kinematic constraint.

At $d = 3$, the Fourier transform of the function (6) oscillates at large r,

$$U(\rho) \sim -\frac{U_0 s_0^3}{2\pi^2\rho^2}\left(1-\frac{s_0^2}{2}\right)\cos\rho, \qquad (7)$$

and has a finite positive value at $\rho = 0$, where $\rho \equiv k_c r$, $s_0 \equiv k_c r_0$. The magnitude of the oscillation exhibits the maximum at $s_0 = \sqrt{6/5}$ and decreases with r more slowly as compared to Friedel oscillations (3).

In 2D system ($d = 2$), the screened Coulomb potential ensuring the SC η_K - pairing can be estimated (within the framework of an isotropic model) as

$$U(\rho) = \frac{U_0 s_0^2}{2\pi\rho}\left[J_1(\rho)\left(1-\frac{s_0^2}{2}+\frac{2s_0^2}{\rho^2}\right)-\frac{s_0^2}{\rho}J_0(\rho)\right], \qquad (8)$$

where $J_p(x)$ is the Bessel function. The potential (8) has a finite value at $\rho = 0$ and exhibits oscillations slowly damping at large ρ:

$$U(\rho) \sim \frac{U_0 s_0^2}{(\pi\rho)^{3/2}}\left(1-\frac{s_0^2}{2}\right)\sin\rho. \qquad (9)$$

The magnitude of distant oscillations of the potential (8) peaks at $s_0 = 1$.

The potential (8) is shown schematically in fig.3 at $s_0 = 1$. Note that this potential exhibits more deep (at the same value of the parameter s_0) oscillations as compared to three-dimensional potential (7).

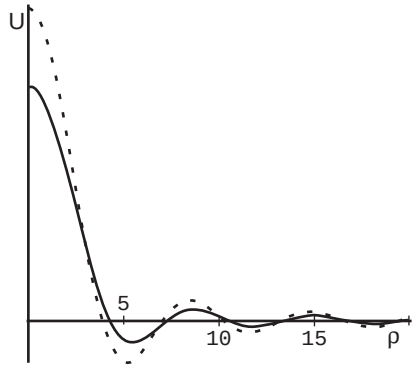

Рис. 3: 2D pairing potential (8) and step-wise (in the momentum space) potential (11) (in arbitrary units, solid and dashed lines, respectively).

Since the cutoff momentum k_c, which defines the pairing interaction (6), increases with doping, the dependence of the kernel magnitude on s_0 (initial increase up to the maximum and the further decrease) can be considered as one of the reasons resulting in typical of cuprates dependence of the transition temperature on doping, $T_c(x)$.

The extension of the hole pockets (an increase in the FC length) with doping promotes a rise in T_c due to the increase of carrier concentration but, on the other hand, leads to the decrease in the spectral weight of the shadow band of the FC and, as a result, the lowering of T_c.

6. The elimination of all Fourier components of the screened Coulomb potential that do not belong to the domain Ξ results in the fact that, in the real space, the pairing potential exhibits deep damped oscillation with the wavelength π/k_c. Precisely this feature of the screened Coulomb repulsion becomes apparent in the η_K - pairing channel. In this connection, one can remind the change in Coulomb repulsion, arising within the framework of the electron-phonon model of superconductivity, due to the dynamic constraint of the region of effective attraction in the vicinity of the Fermi surface,[13]

$$U_0 \to U_0/[1+gU_0\ln(E_F/\varepsilon_D)], \qquad (10)$$

where E_F is the Fermi energy, ε_D is the characteristic Debye energy.

It should be noted that the change of $U(k)$ by a positive constant U_0 inside Ξ leads only to the trivial ($\Delta = 0$) solution to the self-consistency equation in spite of the fact that the corresponding potential

$$U(\rho) = \frac{U_0 s_0^2}{2\pi\rho}J_1(\rho) \qquad (11)$$

oscillates in the real space. Therefore, oscillation of $U(\rho)$ is a necessary but not yet sufficient condition of the nontrivial solution existence. Such a solution exists if and only if the kernel $U(\boldsymbol{k}-\boldsymbol{k}')$ of the pairing interaction has

at least one negative eigenvalue.[7] It is obvious that the kernel $U = U_0$ inside Ξ and $U = 0$ outside of Ξ has only one positive eigenvalue.

Degenerate kernel (6) that approximates the screened Coulomb repulsion at small momentum transfers (important in the case of η_K - pairing) results in four eigenfunctions (two even and two odd with respect to the transformation $\boldsymbol{k} \to -\boldsymbol{k}$) even if r_0 is arbitrarily small. A negative eigenvalue corresponds to one of the even eigenfunctions. A rise of the negative eigenvalue at the transition from a step-wise kernel to kernel (6) can be compared with the well-known problem of quantum mechanics related to one-dimensional motion of a particle in asymmetric potential well:[14] the change in the parameters of the kernel lowering the degree of asymmetry of the potential (11) leads to splitting off the discrete level from the band of continuous spectrum.

7. Due to the smallness of hopping integrals between cuprate layers (labelled by $\mu = 1, 2 \ldots, n$), the Fermi surface proves to be opened along the k_z - axis corresponding to the c - axis of the unit cell. The k_x, k_y - sections of the Fermi surface corresponding to the set of layers in the unit cell present the set of the FC's defined in the sections with different μ. Since the Brillouin zone size along k_z is $2\pi/c$, where c is the size of the unit cell along the z - axis, the characteristic separation between neighboring sections can be estimated as $k_n = 2\pi/nc$.

The characteristic size of the domain of kinematic constraint $k_c \ll \pi/a$, where a is the period of the cuprate plane, determines the wavelength of the pairing potential oscillation in this plane. It is natural to assume that the momentum k_0, which determines the radius of action of the screened Coulomb potential in normal to the layers direction, exceeds k_c.

This permits of a possibility of SC pairing with the momentum $\boldsymbol{K}$ not only in the case when the momenta of the particles $\boldsymbol{k}_\pm$, $\boldsymbol{k}'_\pm$ (before and after scattering due to the pairing interaction, respectively) belong to the same cuprate plane (fig.4a) but also when these momenta are related to different nearest neighboring planes (fig.4b). Another possibility of an elevation of T_c, considered in Ref.[5], is connected with the tunnelling of pairs between the neighboring planes: the momenta $\boldsymbol{k}_\pm$ of particles before scattering belong to one plane whereas the momenta $\mathbf{k}'_\pm$ after scattering belong to the neighboring one (fig.4c).

The enhancement of efficiency of the η_K - pairing channel leads to the natural explanation of the dependence of T_c on the number n of cuprate planes in the unit cell typical of the homologous cuprate series.

8. Let us designate the energy gap parameter relating to the μ-th section of the Brillouin zone by $\Delta_\mu(\boldsymbol{k})$. In the case of n - layer compound, the self-consistency equation can be written in the form of a system of quasi-linear integral equations

$$\Delta_\mu(\mathbf{k}) = -\frac{1}{2}\sum_{\mu'=1}^{n}\sum_{\mathbf{k}'} A_{\mu\mu'}(\mathbf{k},\mathbf{k}';T)\Delta_{\mu'}(\mathbf{k}'), \qquad (12)$$

where the sum in the right-hand side is over all sections

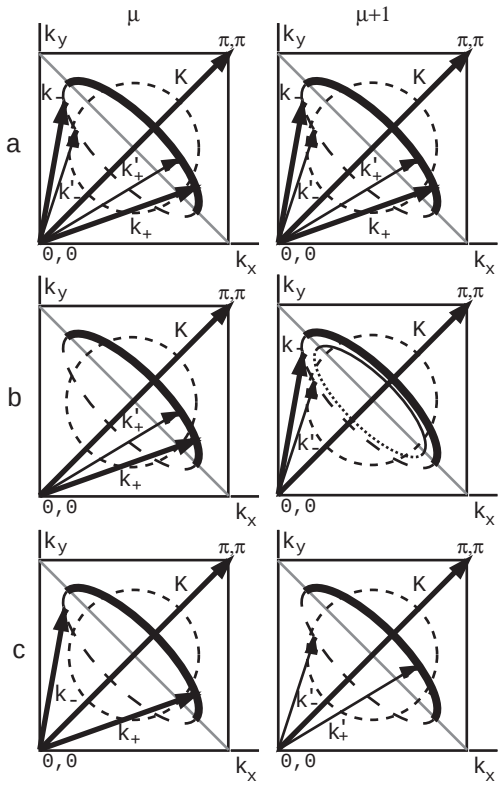

Рис. 4: SC pairing with large momentum $\boldsymbol{K}_\pi = (\pm\pi, \pm\pi)$. **a:** The momenta of particles composing η_K - pair before ($\boldsymbol{k}_+$ and $\boldsymbol{k}_-$) and after ($\boldsymbol{k}'_+$ and $\boldsymbol{k}'_-$) scattering belong to the same cuprate plane. **b:** One of the particles with the momenta $\boldsymbol{k}_+$ and $\boldsymbol{k}'_+$ after and before scattering, respectively, is in the layer μ, and the other particle with the corresponding momenta $\boldsymbol{k}_-$ and $\boldsymbol{k}'_-$) is in the layer $\mu+1$. **c:** Both particles of the pair with the momenta $\boldsymbol{k}_+$ and $\boldsymbol{k}_-$ before scattering being in the layer μ pass (with the momenta $\boldsymbol{k}'_+$ and $\boldsymbol{k}'_-$) into the layer $\mu+1$. The nodal line of the SC order parameter is shown by dashed line. In fig.4b is shown a change of the FC (fine solid and dashed lines correspond to the main and shadow bands, respectively) due to the nonhomogeneous doping of the layers.

of the Brillouin zone taking into account the fact that $\mathbf{k} \in \Xi_\mu$ in each section. The kernel of this system, which depends on $\Delta_{\mu'}(\mathbf{k}')$ and temperature T, has the form

$$A_{\mu\mu'}(\mathbf{k},\mathbf{k}';T) = \frac{U_{\mu\mu'}(\mathbf{k},\mathbf{k}')}{E_{\mu\mu'}(\mathbf{k}')}\tanh\frac{E_{\mu\mu'}(\mathbf{k}')}{2T}. \qquad (13)$$

Generally speaking, the FC's do not coincide in different sections of the Brillouin zone due to the different doping levels of the layers in the unit cell. For the same reason, the quantities

$$E_{\mu\mu'}(\mathbf{k}) = \sqrt{\xi^2_{\mu\mu'}(\mathbf{k}) + \Delta^2_{\mu'}(\mathbf{k})}, \qquad (14)$$

do not coincide as well. Here, $2\xi_{\mu\mu'}(\mathbf{k}) = \varepsilon(\mathbf{K}/2 + \mathbf{k}) + \varepsilon(\mathbf{K}/2 - \mathbf{k})$ is the kinetic energy of two particles in the layers μ and μ', respectively.

The nontrivial solution to self-consistency equation system (13) under pairing repulsion has to be the function alternating sign inside the domains Ξ in each section of the Brillouin zone: the energy gap parameter changes its sign on the nodal line (circumference) intersecting the FC (fig.4). The fact that, in the η_K - pairing scheme, the SC order parameter has a nodal line can be associated with the restriction to double occupation of sites of the cuprate planes taken into account under the choice of the ground state wave function in terms of either Gutzwiller[15] or gossamer[16] projection operators.

For estimation, it is convenient to approximate $\Delta_\mu(\boldsymbol{k})$ by step-wise functions which have the meaning of average values of the function $\Delta_\mu(\boldsymbol{k})$ in the regions where they are of constant signs. Thus, $\Delta_\mu(\boldsymbol{k})$ can be defined by the constants:[17] $\Delta_\mu(\boldsymbol{k}) = \Delta_\mu(1)$, if $\boldsymbol{k}$ belongs to the part of Ξ inside the nodal line, and $\Delta_\mu(\mathbf{k}) = \Delta_\mu(2)$, if $\boldsymbol{k}$ is outside this line.

Such an approximation allows us to reduce the kernel $U(\boldsymbol{k}-\boldsymbol{k}')$ of the pairing interaction to the degenerate step-wise kernel that can be defined by three constants[17] $U(11)$, $U(22)$, and $U(12)$. Two of the constants, $U(11)$ and $U(22)$, describe scattering inside and outside the nodal line, respectively. The third constant, $U(12)$, describes scattering between the regions of the momentum space separated by the nodal line. Such a kernel complies with the Suhl inequality[18] $[U(12)]^2 > U(11)U(22)$ ensuring the SC pairing under repulsion.

The pairing interaction (5) can be reduced into a set of constants $U_{\mu\mu'}(\alpha\alpha')$ where $\alpha, \alpha' = 1, 2$. The approximation we use here allows to transform the system of integral equations (12) into the system of transcendental equations,

$$\Delta_\mu(\alpha) = -\frac{1}{2}\sum_{\mu'=1}^{n}\sum_{\alpha'=1}^{2} U_{\mu\mu'}(\alpha,\alpha') f_{\mu\mu'}(\alpha';T)\Delta_{\mu'}(\alpha'), \tag{15}$$

where

$$f_{\mu\mu'}(\alpha;T) = \sum_{\mathbf{k}\in\Xi^\alpha_{\mu'}} \frac{\tanh\left[E_{\mu\mu'}(\alpha;\mathbf{k})/2T\right]}{E_{\mu\mu'}(\alpha;\mathbf{k})}, \tag{16}$$

$\Xi^\alpha_{\mu'}$ is the part of the domain of the kinematic constraint inside ($\alpha = 1$) or outside ($\alpha = 2$) the nodal line,

$$E_{\mu\mu'}(\alpha;\mathbf{k}) = \sqrt{\xi^2_{\mu\mu'}(\mathbf{k}) + \Delta^2_{\mu'}(\alpha)}. \tag{17}$$

The mean-field SC transition temperature is determined by the condition that $T \to T_c - 0$ when $\Delta_\mu(\alpha) \to 0$.

9. To analyze equation system (15), we use the simplest approximation[17] consistent with the Suhl inequality, namely,

$$U_{\mu\mu'}(\alpha,\alpha) = 0,\ U_{\mu\mu}(12) = 2w_0,\ U_{\mu\mu\pm1}(12) = 2w_1. \tag{18}$$

Thus, the pairing in the same plane (fig.4a) is described by the coupling constant w_0 whereas another coupling constant, w_1, is related to the pairing in the nearest neighboring planes (fig.4b,c). Note that, in the case of tunnel mechanism of the interlayer pairing (fig.4c; such a mechanism is considered by Chakravarty et al.[5]), $w_1 \ll w_0$. On the contrary, in the case when the momenta of the particles before and after scattering belong to the neighboring sections of the Brillouin zone (fig.4b), it is natural to think that both constants, w_0 and w_1, are of the same exponent.

At $T \to T_c - 0$, parameters (16) can be represented as

$$f_{\mu\mu'}(\alpha;T_c) = g\int_0^{\varepsilon_0} \tanh\left(\frac{\xi}{2T_c}\right)\frac{d\xi}{\xi} = g\ln\left(\frac{2\gamma\varepsilon_0}{\pi T_c}\right), \tag{19}$$

where g is the density of states per one spin, ε_0 is the energy scale of the domain of kinematic constraint Ξ^α_μ, and $\ln\gamma = 0.577$ is the Euler constant. Due to the logarithmical dependence of the quantities (16) on ε_0 one can use mutual energy scale for each of the domains Ξ^α_μ.

In the case of one-layer compound, equation system (15) leads to the typical of the mean-field theory relation between the transition temperature and the coupling constant,

$$T_c(1) = (2\gamma\varepsilon_0/\pi)\cdot\exp\left(-1/gw_0\right). \tag{20}$$

Within the approximation we use, two components of the energy gap parameter have equal absolute values and are of opposite sign: $\Delta_1(1) = -\Delta_1(2)$.

Equation system (15) describing two-layer compound has two solutions for unknown quantity $f \equiv g\ln(2\gamma\varepsilon_0/\pi T_c)$: $f_\pm = (w_0 \pm w_1)^{-1}$. The solution with the upper sign is related to more high, as compared to $T_c(1)$, transition temperature,

$$T_c(2) = (2\gamma\varepsilon_0/\pi)\cdot\exp\left[-1/g(w_0+w_1)\right], \tag{21}$$

which is determined by the effective coupling constant $w_2^* = w_0 + w_1$. The sign distribution of the energy gap parameter is the same in both planes.

As it follows from (21), tunnel coupling between cuprate planes (fig.4c), when $w_1 \ll w_0$, cannot ensure a significant increase in the transition temperature. Therefore, we can restrict ourselves to the examination of the pairing mechanisms shown in fig.4a,b. For the sake of simplicity, we also put $w_1 = w_0$. This approximation leads to the doubling of the effective coupling constant, $w_2^* = 2w_0$ in the case $n = 2$.

Such an approximation would have led to the tripling of the coupling constant in the case of three-layer compound. However, the restriction due to account of the interaction between only the nearest neighboring layers results in the deceleration of the growth of the effective coupling constant: the inner layer has two neighboring layers whereas each of the outer layers has only one neighboring layer. At $n = 3$, this constant turns out to be equal to $w_3^* = 2.247w_0$. When n increases, the effective coupling constant manifests a gradual saturation. In particular, $w_4^* = 2.252w_0$.

The choice of the kernel of the pairing interaction under the complementary condition $U_{\mu\mu'}(\alpha\alpha) = 0$ corresponding to the symmetry between the filled and vacant parts of Ξ does not affect a qualitative estimate of the dependence $T_c(n)$. An important effect of the asymmetry of the domain of kinematic constraint is a chemical potential shift arising at the transition into the SC state. This shift appears to be proportional to the first power of the gap absolute value and may be related to the problem of high-energy states[3] that become apparent in optical conductivity as optical sum rule violation.[20]

10. The nonhomogeneity of cuprate layer doping in the unit cell (which is essential in the case $n \geq 3$) results in the violation of the mirror nesting condition for interlayer pairing interaction (fig.4b). This happens since the unequal hole concentration in different layers. Thus, the FC's in the corresponding sections of the Brillouin zone should be also different (fig.4b).

In this sense, the difference of carrier concentrations in the neighboring layers is similar to the exchange field in SC weak ferromagnets resulting in the Fulde-Ferrel-Larkin-Ovchinnikov state[21,22] with non-zero (small) condensate momentum. For this reason, one can expect a small difference between the condensate momentum and $\mathbf{K}_\pi$ in the compound with $n \geq 3$.

A deviation from mirror nesting (in the case of interlayer pairing) eliminating the logarithmic singularity in the self-consistency equation results in the effective decrease in T_c in the neighboring layers (which, furthermore, differs from the optimum T_c for the one-layer compound of the homologous series). Deviations from the mirror nesting and optimum doping can be considered as the main reasons for a rise in falling branch of the function $T_c(n)$ at $n \geq 3$. These reasons should be strengthened by electrostatic effects when n increases.

11. It is clear that equation system (15) corresponding to the weak-coupling approximation is inadequate for the evaluation of T_c because the Coulomb coupling constant w_0 cannot most likely be considered as a small quantity. More realistic estimation should be obtained within the framework of a renormalization scheme similar to the McMillan's (developed[23] for phonon-mediated pairing) approach: $w_0 \to w_0/(1 + w_0)$.

It should be noted in this connection that an increase in the coupling constant by itself does not result in the considerable growth of T_c. The latter depends essentially on the preexponential factor ε_0 (according to the phonon-mediated pairing mechanism, ε_0 coincides with a characteristic Debye energy of several hundreds K). Under the η_K - pairing, ε_0 is determined by the energy scale of the domain of kinematic constraint. In the case of cuprates with the energy band of about of one eV it can exceed 1000 K in spite of relative smallness of this domain. Thus, within the framework of such a pairing scheme at reasonable values of the coupling constant $w_0 g \leq 1$, there are no apparent grounds excluding the possibility of the transition temperature $T_c \approx 300K$ in 2D compounds with mirror nesting of the FC.

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